

LABORATORIYA ISHI №15
MAVZU: GRAFLAR NAZARIYASI, GRAFNING TASVIRLANISHI.
GRAFLAR IZOMORFIZMI.

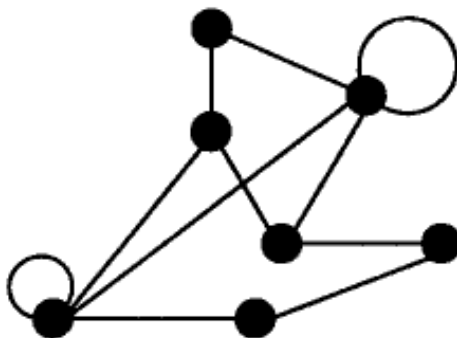
Ishning maqsadi: Talabalarda graflar nazariyasi, grafning tasvirlanishi bo'yicha ko'nikma va malakalarini shakllantirish.

Nazariy qism:

Graflar nazariyasi – bu diskret matematikaning sohasi bo'lib, ayniqsa geometrik yondashuv asosida obyektlarni o'rganishni bildiradi. Odatda, graflarni topologiyaga kiritish mumkin, lekin ular juda ko'p fanlarda ham uchraydi. Graflar nazariyasining Birinchi masalalari turli xil mantiqqa oid masalalar bo'lgan.

$G = (V, E)$ graf (tarmoq) – G grafning qirralari deb ataluvchi bo'sh bo'lmagan ($m \geq 1$) m uchlar to'plami va tartiblanmagan $[u, v](n \geq 0)$ juft elementlar to'plamidan tuzilgan.

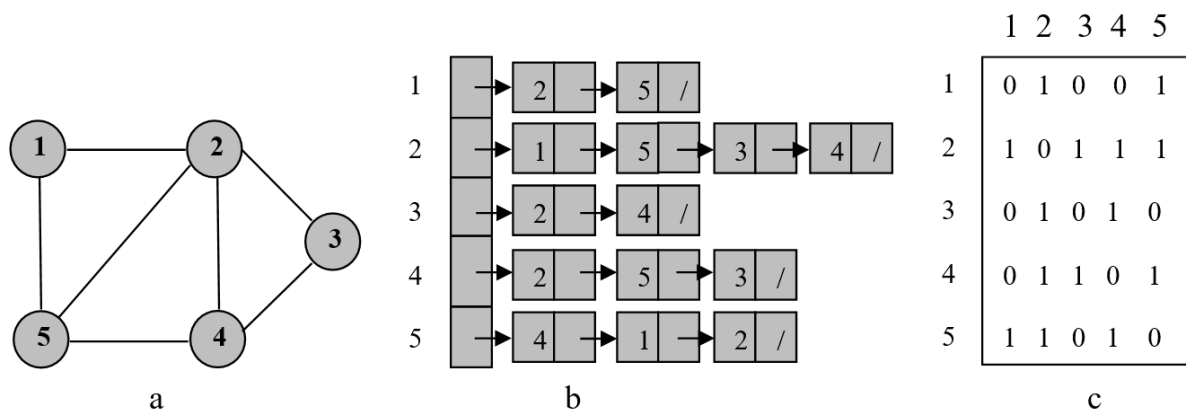
Uchlarni o'zi bilan bog'lab turuvchi qirraga halqa deyiladi. Agar G da (u, v) qirra mavjud bo'lsa, u holda ikkita u va v uchlar qo'shni bo'ladi:



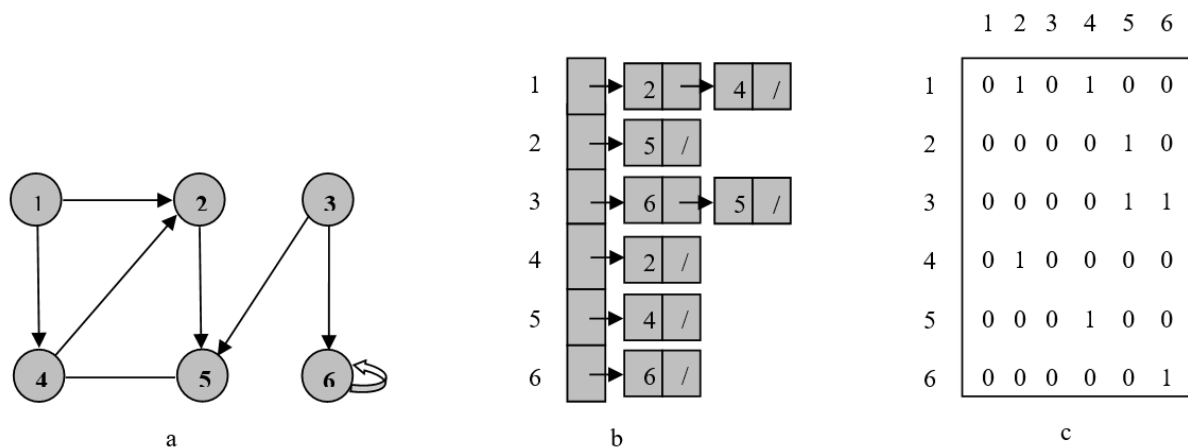
15.1-rasm. Grafli tuzilish (tarmoq).

Grafning tasvirlanishi. $G = (V, E)$ graflarni ikki xil usulda tasvirlash mumkin: uchlar ro'yhatlari jamlanmasi va qo'shni matrisalar. Ikki usul ham orientirlangan va orientirlanmagan graflarda qo'llaniladi. Ko'p hollarda uchlar ro'yhatlari jamlanmasi foydalaniladi, chunki bu usul ruxsat etilgan graflarni $|E| V^2$ dan kichik bo'lgan hollarda ixchamroq tasvirlaydi. Oriyentirlangan graf- bu qirralarida harakat yo'nalishlari bo'yicha garf o'qlari joylashgan, boshqacha qilib aytganda graf o'qlariga strelkalar qo'yilgan bo'ladi. Biz odatdagi graflarda fundamental tushunchalar- uch darajalari, sikl va yo'nalishlar kabi komponentlarni bo'qliq deb qaraymiz. Qo'shni matritsalar orqali tasvirlash ikkita uchlarini

bog'lovchi qirralarni topish hamda qalin graflar uchun samaraliroqdir.



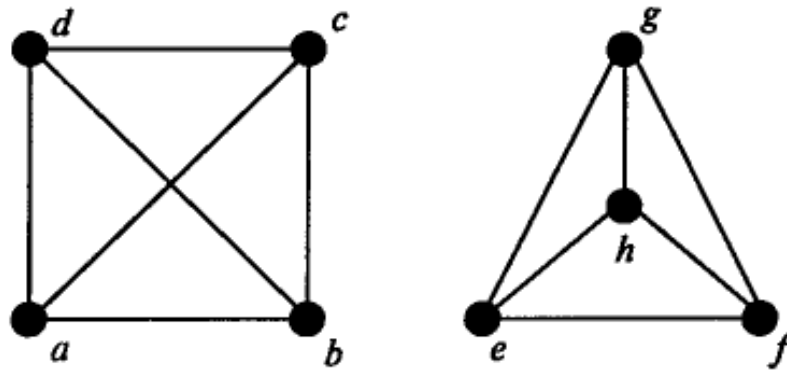
15.2-rasm. Ikkita oriyentirlanmagan graf tasviri a) G beshta uch va sakkiz qirrali oriyentirlangan graf. b) G qo'shni ro'yhatlar yordamida tasvirlangan c) G qo'shni matrisalar yordamida tasvirlangan.



15.3-rasm. Ikkita oriyentirlangan graf tasviri a) G olti uch va sakkiz qirrali oriyentirlangan graf. b) G qo'shni ro'yhatlar yordamida tasvirlangan c) G qo'shni matrisalar yordamida tasvirlangan.

Graflar izomorfizmi. Izomorfizm tushunchasi – graflar uchun alohida o'ringa ega, graf uchlari uchlari tugunlarini bog'lab turuvchi elastik iplar bilan bog'langan deb faraz qilamiz.

Quyidagi 2 ta graflar izomorf ekanligini ko'rsatamiz (15.4-rasm).



15.4-rasm. Izomorf graflar

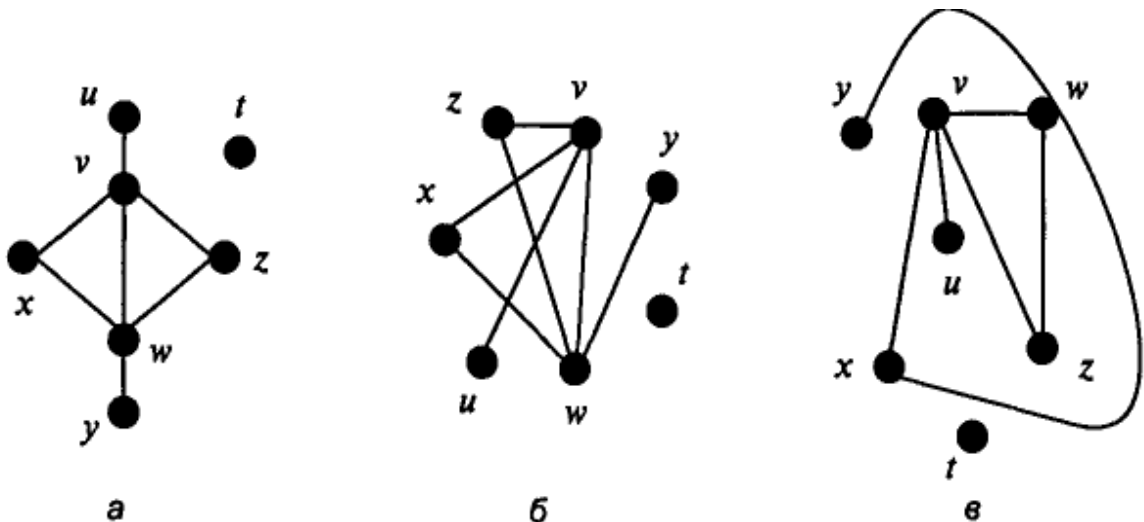
Haqiqatan ham, $a \rightarrow e, b \rightarrow f, c \rightarrow g, d \rightarrow h$ izomorf akslantirishda birinchi graf d-uchidan rasm markaziga siljishini ko‘rishimiz mumkin.

Graf uchlari darajasi

V graflar uchlari darajasini bildiruvchi d_v yoki $\deg(v)$ uning qirralar soniga teng bo‘ladi, ya’ni

$$d_v = \deg(v) = |N(v)|$$

Shuningdek, agar G_1 ning uchlari w,v,x,z,u,y,t tartibda joylashgan bo‘lsa, u holda ularga mos. 4,4,2,2,1,1,0 ya’ni $d_w=4, d_v=4, d_x=2$ va hokazo darajalar mavjud.

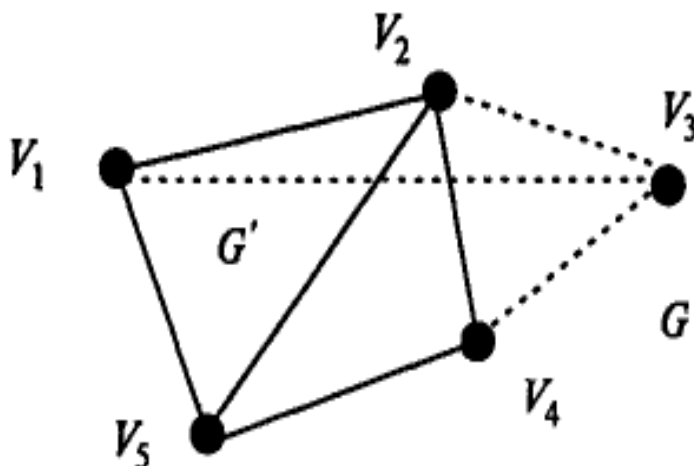


15.5-rasm. Tarmoq izomorfizmi

Graf uchi darajasini aniqlashda halqa ikki marotaba hisobga olinadi. Agar $\deg(v)=1$ ga teng bo‘lsa, u holda V **uch qulflangan** deyiladi. Agar $\deg(v) = 0$ ga teng bo‘lsa, u holda v-**izolyatsiyalangan uch** deyiladi.

Qism graf tushunchasi. Agar $V' \subseteq V$, va $E' \subseteq E$ bo‘lsa, u holda $G' = (V', E')$ tarmoq $G = (V, E)$ tarmoqning qism tarmog‘i bo‘ladi. Agar $v \in V$ bo‘lsa, u

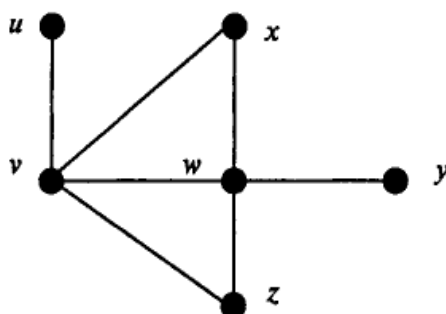
holda uchlari $-\{v\}$, qirralari E dagi barcha qirralardan iborat $G-v$ qismtarmoqni bildiradi.



15.6-rasm.

$G = (V, E)$ grafning u_1 uchidan u_n yo‘nalish- bu har bir $i, 1 \leq i \leq n - 1$ uchun $(u_i, u_{i+1}) \in E(G)$ bo‘lgan $W = u_1, u_2, \dots, u_n$ uchlar ketma-ketligidir. Agar $u_1 = u_n$ bo‘lsa, u holda W yo‘nalish yopiq, boshqa hollarda esa ochiq bo‘ladi. agar bitta ham uch W bir martadan ko‘p payda bo‘lmasa.

15.7-rasmda $uvxwywzv$ yo‘nalish bo‘lib; $uvxwz$ – yozuv yo‘l, $vxwzv$ – sikl. Har bir qirra juftliklari turli xil bo‘lgan yo‘nalishlar zanjir deb ataladi. Yopiq yo‘nalish sikl deb ataluvchi zanjir hisoblanadi.



15.7-rasm. Graflarda yo‘nalish va sikl.

Har bir uchlar juftliklari turli xil bo‘lgan yo‘nalish oddiy zanjir deb ataladi. birinchi va oxirigidan tashqari, Barcha uchlar juftliklari turli xil bo‘lgan sikl, oddiy sikl deyiladi.

LABORATORIYA ISHINI BAJARISH UCHUN NAMUNA:

1-misol. Quyidagi $G=(V,E)$ grafni ko‘rib chiqamiz:

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\},$$

$$E = \{e_1 = (v_1, v_3), e_2 = (v_3, v_2), e_3 = (v_4, v_1), e_4 = (v_4, v_5), e_5 = (v_5, v_3),$$

$$e_6 = (v_6, v_5), e_7 = (v_6, v_5), e_8 = (v_2, v_1)\}.$$

Ushbu graf rasmda keltirilgan, u uchun yuqorida keltirilgan ta’rifni ko‘rib chiqamiz.

1. Qo‘shni matrisa

$$A_G = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2. Insidentlik matrisasi

$$B_G = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \end{pmatrix}$$

3. Qo‘shni ro‘yhatlar

$$L_{v_1} : v_3 :$$

$$L_{v_2} : v_1 :$$

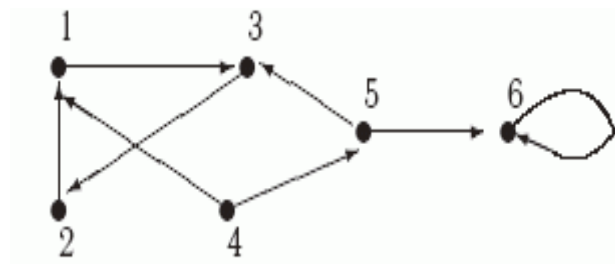
$$L_{v_3} : v_3 :$$

$$L_{v_4} : v_3, v_5 :$$

$$L_{v_5} : v_3, v_6 :$$

$$L_{v_6} : v_6.$$

2-misol. G graf uchun A_{G^*} erkli graf matrisasini rasmda ko‘rsatilgan misoli tariqasida hisoblashni ko‘rib chiqamiz.



Bu holda

$$\tilde{A} = A_G + E_8 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

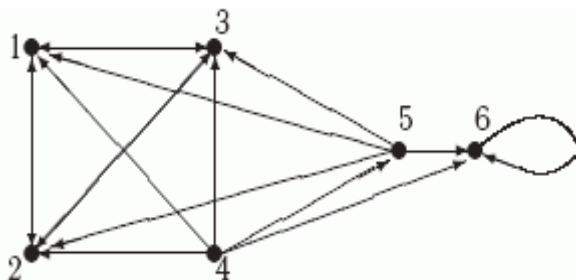
G da 6 ta uch bo'lganligi uchun $A_{G^*} = \tilde{A}^5$ bo'ladi. Ushbu matrisani hisoblaymiz:

$$\tilde{A}^2 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \tilde{A}^4 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

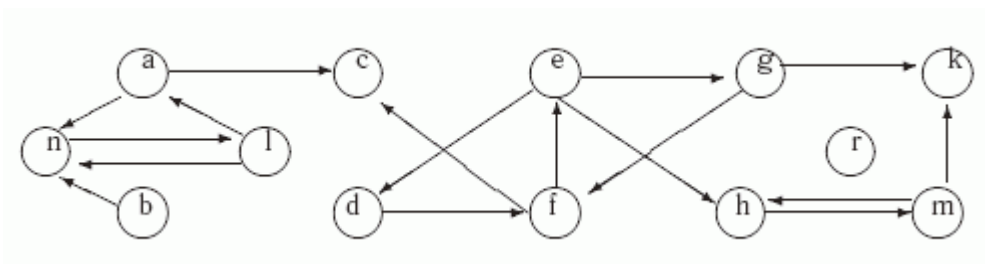
va $\tilde{A}^5 = \tilde{A}^4 \times \tilde{A}^4 = \tilde{A}^4$. Shu tariqa,

$$A_{G^4} = \tilde{A}^4 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Ko‘rinib turibdiki, ushbu matrisa quyida keltirilgan G^* grafni beradi.



3-Misol. Rasmda keltirilgan barcha oriyentirlanmagan G graf bazasini aniqlaymiz.



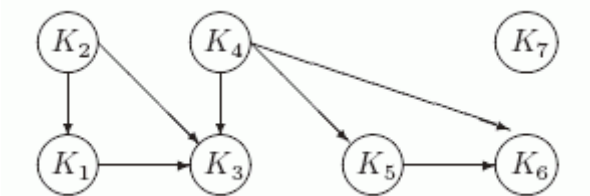
G graf

Birinchi bosqichda G bilan kuchli bog'langan komponentalarni topamiz.

$$K_1 = \{a, n, l\}, K_2 = \{b\}, K_3 = \{c\}, K_4 = \{d, e, f, g\},$$

$$K_5 = \{h, m\}, K_6 = \{k\}, K_7 = \{r\}$$

Ikkinchi bosqichda ushbu komponentlar bilan qat'iy erklilik grafni quramiz



Eng kichik komponentni topamiz: $K_2 = \{b\}$, $K_4 = \{d, e, f, g\}$ va $K_7 = \{r\}$.

Nihoyat, jami 4 ta bazani sanab o'tamiz:

G: $B_1 = \{b, d, r\}$, $B_2 = \{b, e, r\}$, $B_3 = \{b, f, r\}$ va $B_4 = \{b, g, r\}$.

LABORATORIYA ISHINI BAJARISH UCHUN TOPSHIRIQLAR:

1. $M = \{1, 2, 3, 4, 5, 6\}$ to'plamda $x + y \leq 7$ munosabatdagi grafni quring. Uning xossaarini aniqlang.
2. Uchlari 4 dan ko'p bo'lmagan barcha izoforb bo'lmagan oriyentirlanmagan graflarni sanang.
3. n uchli oriyentirlanmagan bog'langan graf :
 - a) $n-1$ kam bo'lmagan qirradan;
 - b) agar $n-1$ qirradan ko'p bo'lsa, u holda kamida bitta sikl mavjud ekanligini isbotlang.
4. Olti kishidan iborat ixtiyoriy guruhda 3 ta tanishlar juftliklar yoki 3 ta notanish juftliklar mavjudligini isbotlang.
5. Agar oriyentirlanmagan grafda ikkita toq darajali uchlar teng bo'lsa, u holda ular yo'nalish bo'yicha bog'liq.
6. $|E| < |V| - 1$ bo'lgan $G = (V, E)$ oriyentirlanmagan graf bo'lsin. G bog'lanmagan graf ekanligini isbotlang.

7. A_{G^4} erkli matrisa grafi uchun

$$G = (\{a, b, c, d, e, f, g\}, \{(a, b), (b, a), (a, c), (b, d), (e, d), (d, f), (f, c), (c, f), (g, e)\})$$

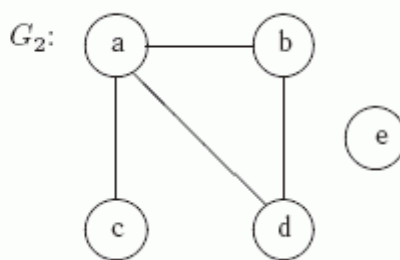
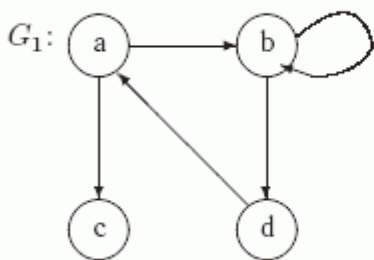
graf va unga mos erkli grafni quring. G grafning barcha bazalarini toping.

8. Qo'shni ro'yhatdan iborat oriyehtirlangan graf tasavvuri mavjud. Qancha vaqtda grafning barcha uchlari chiqish darajalari hisoblanadi? Kiruvchi graflarchi?

9. Qo'shni ro'yhatdan foydalanilgan 7 ta uchdan iborat binar daraxt tasavvuri mavjud. Qo'shni matrisalardan foydalanib, ularni tasvirini keltiring.

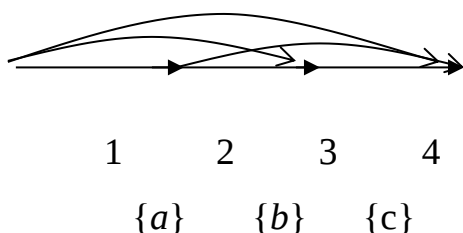
10. $G = (V, E)$ oriyehtirlangan grafni transponirlashdan (**transpose**) $E^T = \{(u, v) \in V \times V : (v, u) \in E\}$ bo'lgan $G^T = (V, E^T)$ grafni hosil qilamiz. Qo'shni ro'yhatlardan foydalanib, transponirlangan grafning samarali algoritmini tasvirlang. Algoritmning ish vaqtini tahlil qiling.

11. Rasmda $G_1 = (V_1, E_1)$ orientirlangan graf va $G_2 = (V_2, E_2)$ orientirlanmagan graf keltirilgan. Bu yerda $V_1 = \{a, b, c, d\}$, $E_1 = \{(a, b), (a, c), (b, b), (b, d), (d, a)\}$, $V_2 = \{a, b, c, d, e\}$, $E_2 = \{(a, b), (a, c), (a, d), (b, d)\}$. G_1 grafda (b, b) qirra uning halqasi hisoblanadi, berilgan a uchning yarim darajasi 2 ga teng, a ning yarim kirish darajasi 1 ga teng. G_2 grafda a uchning darajasi 3 ga, b va d uchlar- 2, c uch 1 ga, e esa 0 ga teng va hokazo bo'lib.



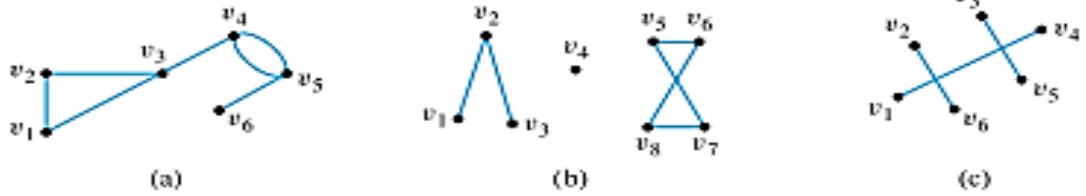
G_1 – oriyehtirlangan graf va G_2 . oriyehtirlanmagan graf

12. R - binar munosabat $A = \{1, 2, 3, 4\}$ to'plamdagi «<» munosabat bo'lsin. U holda «<» munosabatni graf yordamida quyidagi ko'rinishda ifoda qilish mumkin:



13. Qo'shni ro'yhatlardan foydalanilgan, $G = (V, E)$ multigraf tasavvuri mavjud. Oriyentirlanmagan $G' = (V, E')$ graf va qo'shni ro'yhatlar bilan "Ekvivalent" E qirradan iborat E' algoritmini tasvirlang uchun $O(V + E)$ ish vaqti algoritmini yozing.

14. Quyidagi graflarning qaysi biri bog'langan.



15. Quyidagi rasmlarning bir xil ekanligini ko'rsating.

Edge	Endpoints
e_1	$\{v_1\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_2, v_3\}$
e_4	$\{v_2, v_5\}$